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Monopole operators in three-dimensional $\mathcal{N} = 4$ SYM and mirror symmetry

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Abstract: We study non-abelian monopole operators in the infrared limit of three-dimensional SU(N_c) and $\mathcal{N} = 4$ SU(2) gauge theories. Using large N_f expansion and operator-state isomorphism of the resulting superconformal field theories, we construct monopole operators which are (anti-)chiral primaries and compute their charges under the global symmetries. Predictions of three-dimensional mirror symmetry for the quantum numbers of these monopole operators are verified.

Keywords: Nonperturbative Effects, Solitons Monopoles and Instantons, Duality in Gauge Field Theories, Supersymmetry and Duality.

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JHEP03(2004)008

Contents

1. Introduction

A non-perturbative duality in three dimensions is known as 3d mirror symmetry. It was first proposed by K. Intriligator and N. Seiberg in ref. [1] and further studied in refs. [2]–[18]. The mirror symmetry predicts quantum equivalence of two different theories in the IR limit. In this regime a supersymmetric gauge theory is described by a strongly coupled superconformal field theory. The duality exchanges masses and Fayet-Iliopoulos terms as well as the Coulomb and Higgs branches implying that electrically charged particles in one theory correspond to the magnetically charged objects (monopoles) in the other. Also, since the Higgs branch does not receive quantum corrections and the Coulomb branch does, mirror symmetry exchanges classical effects in one theory with quantum effects in the dual theory. Many aspects of three-dimensional mirror symmetry have a string theory origin.

This paper extends the analysis of refs. [19, 20] to the non-abelian gauge theories. We study monopole operators in the IR limit of SU(N_c) non-supersymmetric Yang-Mills theories as well as $\mathcal{N} = 4$ SU(2) supersymmetric Yang-Mills models with large number of flavors N_f. Conformal weight of a generic monopole operator in non-supersymmetric gauge theory is irrational. On the other hand, supersymmetric gauge theories have monopole operators which are superconformal (anti-)chiral primaries. The conformal dimensions of such operators are uniquely determined by their R-symmetry representations. The R-symmetry group of $\mathcal{N} = 4$ supersymmetric theory is given by SU(2) × SU(2) and conformal dimensions of the (anti-)chiral primary operators are integral. The mirror symmetry predicts the spectrum and quantum numbers of (anti-)chiral primary operators including the ones with magnetic charges. We use $1/N_f$ expansion and the operator-state isomorphism of the resulting conformal field theories to study transformation properties of monopole operators under the global symmetries and verify the mirror symmetry predictions.

In refs. [21, 22, 23] it was demonstrated that three-dimensional gauge theories have severe perturbative infrared divergences due to logarithms of the coupling constant. In ref. [24] it was shown that for three-dimensional QCD, the $1/N_f$ expansion can be defined in such a way that the infrared divergences are absent in each order of the expansion and the theory has IR fixed point. For large N_f the non-abelian interactions of gluons are suppressed and dynamics of the theory becomes similar to that of an abelian theory. In refs. [24]–[27] it is claimed that for N_f smaller than a certain critical value the dynamical fermion mass is generated. These conclusions were supported by the lattice simulations in ref. [28]. The phase transition takes place at finite N_f and does not affect the dynamics at large N_f which is studied in this paper. However, it indicates that $1/N_f$ expansion has a finite radius of convergence. In the case of $\mathcal{N}=4$ supersymmetric Yang-Mills theories, IR limit of the theory is given by an interacting superconformal field theory and there is no evidence of the phase transitions at finite N_f. It is possible that $1/N_f$ expansion is convergent all the way down to $N_f=1$. Our analysis is performed in the origin of the moduli space, thus extending results of ref. [7], where implications of the mirror symmetry have been verified on the Coulomb branch of $\mathcal{N}=2$ supersymmetric Yang-Mills theories.

The paper is organized as follows. In section 2 we study monopole operators in the IR limit of $\mathrm{SU}(N_c)$ gauge theories and determine their quantum numbers at large N_f. Monopole operators in the IR limit of $\mathcal{N}=4$ SU(2) gauge theories are considered in section 3. We discuss our results in section 4.

2. Monopole operators in $\mathrm{SU}(N_c)$ gauge theories

2.1 IR limit of $\mathrm{SU}(N_c)$ gauge theories

Consider a three-dimensional euclidean Yang-Mills action for N_f flavors of matter fermions in the fundamental representation of the gauge group $\mathrm{SU}(N_c)$ with generators $\{T^\alpha\}$,[1] $(\alpha = 1,\ldots,N_c^2-1)$:

$$S=\int d^3x\left(\frac{1}{4e^2}\mathrm{Tr}\ V_{ij}V^{ij}+i\sum_{s=1}^{N_f}\bar{\psi}^s\vec{\sigma}\left(\vec{\nabla}+i\vec{V}\right)\psi^s\right), \tag{2.1}$$

where ψ are complex two-component spinors, $\vec{V}=\vec{V}^\alpha T^\alpha$ is gauge potential with a field-strength $V_{ij}=\left(\partial_i V_j^\alpha-\partial_j V_i^\alpha-f^{\alpha\beta\gamma}V_i^\beta V_j^\gamma\right)T^\alpha$, $f^{\alpha\beta\gamma}$ are the structure constants. To avoid a parity anomaly, ref. [29], we choose N_f to be even. Action (2.1) is invariant under the flavor symmetry $\mathrm{U}(N_f)_{\mathrm{flavor}}$.

[1] We use $\mathrm{Tr}(T^\alpha T^\beta)=\frac{1}{2}\delta^{\alpha\beta}$ normalization.

JHEP03(2004)008

There are two ways to classify monopoles in non-abelian theories. A dynamical description of monopoles in terms of weight vectors of the dual of (unbroken) gauge group was developed by Goddard, Nuyts, and Olive (GNO) in ref. [30]; topological classification in terms of π_1 was suggested by Lubkin in ref. [31], (see also ref. [32] for a review). It is well known that in $R^{1,3}$ the dynamical (GNO) monopoles with vanishing topological charges are unstable in the small coupling limit. We will study the dynamical monopoles in the IR limit of (2.1). The theory is free in the UV limit ($\frac{e^2}{\Lambda} \to 0$, where Λ is a renormalization scale) and is strongly coupled in the IR ($\frac{e^2}{\Lambda} \to \infty$). In the strong coupling regime the dominant contribution to the gauge field effective action is given by the matter fields and stability analysis of GNO monopoles performed at weak coupling is no longer applicable. Since matter fields belong to the fundamental representation, the effective gauge group is given by $\mathrm{SU}(N_c)$. The corresponding π_1 is trivial and all dynamical monopoles have vanishing topological charges. The GNO monopoles of $\mathrm{SU}(N_c)$ are given by

$$V^N = H(1-\cos\theta)d\varphi\,, \qquad V^S = -H(1+\cos\theta)d\varphi\,, \tag{2.2}$$

where V^N and V^S correspond to gauge potentials on upper and lower hemispheres respectively. H is a constant traceless hermitian $N_c \times N_c$ matrix, which can be assumed to be diagonal. On the equator V^N and V^S are transformed into each other by a gauge transformation with a group element $\exp(2iH\varphi)$. This transformation is single-valued if

$$H = \frac{1}{2}\mathrm{diag}(q_1, q_2, \ldots, q_{N_c}) \tag{2.3}$$

with integers q_a, $(a = 1, \ldots, N_c)$,

$$\sum_{a=1}^{N_c} q_a = 0\,. \tag{2.4}$$

Consider a path integral over matter and gauge fields on the punctured R^3. Integration over the gauge fields asymptotically approaching (2.2) at the removed point of R^3 and corresponds to an insertion of a topology-changing operator with magnetic charge H. To complete definition of the topology-changing operator we have to specify the behavior of matter fields at the insertion point. Thus topology-changing operators with a given magnetic charge are classified by the behavior of the matter fields near the singularity. In the IR limit the theory (2.1) flows to the interacting conformal field theory (CFT). In three-dimensional CFT operators on R^3 are in one-to-one correspondence with normalizable states on $S^2 \times R$. Namely, insertion of a topology-changing operator in the origin of R^3 corresponds to a certain in-going state in the radially quantized theory on $S^2 \times R$. Hamiltonian of the radially quantized theory is identical to the dilatation operator on R^3. In unitary CFT all physical operators including topology-changing ones are classified by the lowest-weight irreducible representations labelled by the primary operators. We will say that topology-changing operator is a monopole operator, i.e., corresponds to the creation of the GNO monopole, if such an operator has the lowest conformal weight among the topology-changing operators with a given magnetic charge H. Since conformal transformations do not affect the magnetic charge, the monopole operators are conformal primaries. Our task is to determine spin, conformal weight and other quantum numbers of the monopole operators.

In the IR limit kinetic term for the gauge field can be neglected and integration over matter fields produces effective action for the gauge field proportional to N_f. Although IR theory is strongly coupled, the effective Planck constant is given by $1/N_f$ and in the large N_f limit the CFT becomes weakly coupled. It is natural to assume that saddle point of the gauge field effective action is invariant under rotations and corresponds to the GNO monopole. Since fluctuations of the gauge field are suppressed, it can be treated as a classical background. Thus, in the large N_f limit we have matter fermions moving in a presence of the GNO monopole. Therefore, a monopole operator is mapped to a Fock vacuum for matter fields moving in a monopole background on $S^2 \times R$. Conformal weight of the monopole operator is equal to Casimir energy of the corresponding vacuum state relative to the vacuum state with vanishing monopole charge.

2.2 Radial quantization

Let us implement the procedure outlined in the previous section. Namely, we consider CFT which appears in the IR limit of the theory (2.1). We neglect the kinetic term of a gauge field, introduce a radial time variable $\tau = \ln r$ and perform the Weyl rescaling to obtain metric on $S^2 \times R$:

$$ds^2 = d\tau^2 + d\theta^2 + \sin^2\theta d\varphi^2 .$$

Since a gauge potential of the GNO monopole (2.2) with H given by eqs. (2.3)–(2.4) is diagonal in color indices we may use results of ref. [19][2] for fermionic energy spectra on $S^2 \times R$. We conclude that for each ψ^s_a, where $s = 1, \ldots, N_f$ and $a = 1, \ldots, N_c$ are flavor and color indices respectively, the energy spectrum is given by

$$E_n = \pm\sqrt{|q_a|n + n^2}\,, \qquad n = 1, 2, \ldots$$

Each energy mode has a degeneracy $2|E_n|$ and spin $j = |E_n| - \frac{1}{2}$. In addition, there are $|q_a|$ zero-energy modes which transform as an irreducible representation of the rotation group $\mathrm{SU}(2)_{\mathrm{rot}}$ with spin $j = \frac{1}{2}(|q_a| - 1)$. In the large N_f limit leading contribution to the conformal weight $h_{\{q\}}$ of the GNO $\mathrm{SU}(N_c)$ monopole is given by

$$h_{\{q\}} = N_f \sum_{a=1}^{N_c} \left(\frac{1}{6}\sqrt{1+|q_a|}\,(|q_a| - 2) + \right. \\ \left. + 4\mathrm{Im}\int_0^\infty dt \left[\left(it + \frac{|q_a|}{2} + 1 \right) \sqrt{\left(it + \frac{|q_a|}{2} + 1 \right)^2 - \frac{q_a^2}{4}} \right] \frac{1}{e^{2\pi t} - 1} \right),$$

where branch of the square root under the integral is the one which is positive on the positive real axis.

Let us specialize in the case of GNO monopole with minimum magnetic charge:

$$H = \frac{1}{2}(1, -1, 0, \ldots, 0)\,, \tag{2.5}$$

and denote the fermionic nonzero-energy mode annihilation operators by a^s_{akm}, b^s_{akm}, where k labels the energy level, and m accounts for a degeneracy. Fermionic zero-energy modes

[2] q in present paper equals twice that in ref. [19].

JHEP03(2004)008

have vanishing spin and are present for ψ_1^s and ψ_2^s only. The corresponding annihilation operators we denote as c_1^s and c_2^s. Consider a Fock space of states obtained by acting with creation operators on a state $|vac\rangle$, which is defined as a state annihilated by all the annihilation operators. Those elements of the Fock space which satisfy the Gauss-law constraints form the physical Fock space.

The background (2.2) with H given by eq. (2.5) breaks gauge group $G = \mathrm{SU}(N_c)$ to $\bar{G} = \mathrm{U}(1)$ for $N_c = 2$ and $\bar{G} = \mathrm{SU}(N_c - 2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ for $N_c > 2$, where generators of the two U(1) groups are given by $(1, -1, 0, \ldots, 0)$ and $(2 - N_c, 2 - N_c, 2, \ldots, 2)$. Let $\bar{T}^{\bar{\alpha}}$ be generators of $\bar{G}$. In a quantum theory we impose Gauss-law constraints on physical states. In the IR limit it implies that they are annihilated by the charge density operators $\rho^{\bar{\alpha}}$. Consider charges $Q^{\bar{\alpha}}$ obtained by integration of $\rho^{\bar{\alpha}}$ over S^2. The most general form of the corresponding quantum operators is

$$Q^{\bar{\alpha}} = Q_+^{\bar{\alpha}} + Q_0^{\bar{\alpha}}\,,$$

where $Q_0^{\bar{\alpha}}$ denote all terms that act within a zero-mode Fock space and $Q_+^{\bar{\alpha}}$ are assumed to be normal-ordered. Using explicit form of zero-energy solutions we find

$$Q_0^{\bar{\alpha}} = c_1^{+s} \bar{T}_{11}^{\bar{\alpha}} c_1^s + c_2^{+s} \bar{T}_{22}^{\bar{\alpha}} c_2^s + n^{\bar{\alpha}}\,,$$

where C-numbers $n^{\bar{\alpha}}$ account for operator-ordering ambiguities. Since zero modes are rotationally invariant, the Gauss-law constraints in the zero-mode Fock space are translated into requirements that the states are annihilated by $Q_0^{\bar{\alpha}}$.

In the case of $N_c = 2$ we have

$$Q_0 = \frac{1}{2}\left(c_1^{+s} c_1^s - c_2^{+s} c_2^s\right) + n\,.$$

The zero-mode space in spanned by the 2^{2N_f} states

$$|vac\rangle\,, \qquad c_1^{+s_1}|vac\rangle\,, \qquad c_2^{+s_1}|vac\rangle\,, \qquad \ldots\,, \qquad c_1^{+s_1} \ldots c_1^{+s_{N_f}} c_2^{+s_1} \ldots c_2^{+s_{N_f}}|vac\rangle\,.$$

A Fock vacuum state as well as completely filled state have Q_0-charge given by n. Since the monopole background is invariant under CP symmetry, we require CP-invariance of the Q_0 spectrum. Therefore, $n = 0$ and we have the following physical vacuum states transforming as scalars under $\mathrm{SU}(2)_{\mathrm{rot}}$

$$|vac\rangle\,, \qquad c_1^{+s_1} \ldots c_1^{+s_l} c_2^{+p_1} \ldots c_2^{+p_l}|vac\rangle\,, \qquad l = 1, \ldots, N_f\,.$$

Each set of the physical vacuum states labelled by l transforms as a product of two rank-l antisymmetric tensor representations under $\mathrm{U}(N_f)_{\mathrm{flavor}}$.

For $N_c > 2$ we choose $\bar{T}^1$ and $\bar{T}^2$ to be generators of the two U(1) groups so that the only zero-mode contributions are

$$Q_0^1 = \frac{1}{2}\left(c_1^{+s} c_1^s - c_2^{+s} c_2^s\right) + n^1\,, \qquad Q_0^2 = -\frac{1}{2}\sqrt{\frac{N_c - 2}{N_c}}\left(c_1^{+s} c_1^s + c_2^{+s} c_2^s\right) + n^2\,.$$

In this case CP-invariance gives $n^1 = 0$ and $n^2 = \frac{1}{2}\sqrt{\frac{N_c-2}{N_c}}N_f$. Therefore, we have $\binom{N_f}{\frac{1}{2}N_f}^2$ physical vacuum states

$$c_1^{+s_1} \dots c_1^{+s_{N_f/2}} c_2^{+p_1} \dots c_2^{+p_{N_f/2}} |vac\rangle \,,$$

transforming as scalars under $\mathrm{SU}(2)_{\mathrm{rot}}$ and as a product of two rank-$N_f/2$ antisymmetric tensor representations of $\mathrm{U}(N_f)_{\mathrm{flavor}}$.

3. Monopole operators in $\mathcal{N} = 4$ SU(2) gauge theory

3.1 IR limit of $\mathcal{N} = 4$ SU(2) gauge theory

Consider three-dimensional euclidean $\mathcal{N} = 4$ supersymmetric theory of vector multiplet $\mathcal{V}$ in the adjoint representation of the gauge group $\mathrm{SU}(2)_{\mathrm{gauge}}$ and N_f matter hypermultiplets $\mathcal{Q}^s$, $(s = 1, \dots, N_f)$, transforming under the fundamental representation. The action in terms of three-dimensional $\mathcal{N} = 2$ superspace formalism is given in the appendix. Decompositions of $\mathcal{N} = 4$ multiplets into $\mathcal{N} = 2$ multiplets are given in the following table

$\mathcal{N} = 4$	$\mathcal{N} = 2$
Vector multiplet $\mathcal{V}$	Vector multiplet $V = (V_i, \chi, \lambda, \bar{\lambda}, D)$, Chiral multiplet $\Phi = (\phi, \eta, K)$.
Hypermultiplet $\mathcal{Q}$	Chiral multiplets $Q = (A, \psi, F)$, $\tilde{Q} = (\tilde{A}, \tilde{\psi}, \tilde{F})$,

where V_i is a vector field in the adjoint representation of the gauge group, χ and ϕ are real and complex adjoint scalars respectively, λ, $\bar{\lambda}$, and η are the gluinos, whereas fields D and K are auxiliary. Scalar A $(\tilde{A})$, spinor ψ $(\tilde{\psi})$, and auxiliary field F $(\tilde{F})$ transform according to (anti-)fundamental representation of the gauge group:

$$Q \to e^{i\omega^\alpha T^\alpha} Q \,, \qquad \tilde{Q} \to \tilde{Q} e^{-i\omega^\alpha T^\alpha} \,,$$

under the gauge transformation with parameters $\omega^\alpha(x)$. Since all representations of SU(2) are pseudo-real, we may define a chiral superfield

$$\Psi^a = \frac{1}{\sqrt{2}} \begin{pmatrix} Q^a - \epsilon^{ab}\tilde{Q}_b \\ i[Q^a + \epsilon^{ab}\tilde{Q}_b] \end{pmatrix},$$

where ϵ^{ab} is antisymmetric tensor with $\epsilon^{12} = 1$. Therefore kinetic term for a hypermultiplet has the form

$$\int d^2\theta d^2\bar{\theta} \sum_{I=1}^{2N_f} \Psi_I^+ e^{2V} \Psi_I \,,$$

where we used the identities

$$\epsilon_{ab} T_c^{\alpha b} \epsilon^{cd} = -(T_d^{\alpha a})^T \,, \qquad \epsilon_{ab}\epsilon^{bc} = \delta_a^c \,.$$

The superpotential is

$$W = i\sqrt{2}\sum_{s=1}^{N_f}\tilde{Q}^s\Phi Q^s = \frac{i}{\sqrt{2}}\sum_{I=1}^{2N_f}\Psi_I^a\epsilon_{ab}\Phi_c^b\Psi_I^c\,.$$

The kinetic term is invariant under $SU(2N_f)$ flavor symmetry. The superpotential, however, is invariant under $SO(2N_f)$ subgroup only.

$4N_f - 6$ dimensional[3] Higgs branch is labelled by the mesons $M_{IJ} = \Psi_I^a\epsilon_{ab}\Psi_J^b$. Using an identity

$$\epsilon^{I_1\ldots I_{2N_f}}\Psi_{I_1}^a\Psi_{I_2}^b\Psi_{I_3}^c\Psi_{I_4}^d = 0\,,$$

we obtain the constraints $\epsilon^{I_1\ldots I_{2N_f}}M_{I_1I_2}M_{I_3I_4} = 0$. The F-flatness condition implies $M_{IJ}^2 = 0$.

On the Coulomb branch adjoint scalars χ and Φ can have nonvanishing expectation values. Let us make a gauge transformation to obtain $\chi = \chi^{(3)}T^3$. Dualizing a photon $V^{(3)} = *d\sigma^{(3)}$ we construct a chiral superfield $\Upsilon = \chi^{(3)} + i\sigma^{(3)} + \cdots$ Potential energy density for scalars χ and Φ is given by $U = U_1 + U_2$, with

$$U_1 \sim \mathrm{Tr}\left(\left[\Phi,\Phi^+\right]\right)^2, \qquad U_2 \sim \mathrm{Tr}\left(\chi^2\right)\mathrm{Tr}\left(\Phi^+\Phi\right) - \left|\mathrm{Tr}\left(\chi\Phi\right)\right|^2.$$

Vanishing of the potential gives $\Phi = \Phi^{(3)}T^3$. Residual gauge symmetries are $U(1)_{\mathrm{gauge}}$ generated by T^3 and Weyl subgroup Z_2 acting by $(\Upsilon,\Phi^{(3)}) \to (-\Upsilon,-\Phi^{(3)})$. Moreover, we have $\Upsilon \sim \Upsilon + 4\pi e^2 i$. Let us introduce a pair of operators Y_+ and Y_- corresponding to positive and negative expectation values of $\chi^{(3)}$ respectively. For large positive (negative) $\chi^{(3)}$ we have $Y_+ \sim e^{\Upsilon/(2e^2)}$ ($Y_- \sim e^{-\Upsilon/(2e^2)}$). We emphasize that none of the $Y_\pm$ is gauge invariant. In fact, $Y_+ \leftrightarrow Y_-$ under the Weyl subgroup Z_2. The gauge invariant coordinates on the Coulomb branch are

$$u = i(Y_+ - Y_-)\Phi^{(3)}\,, \qquad v = (Y_+ + Y_-)\,, \qquad w = \left(\Phi^{(3)}\right)^2. \tag{3.1}$$

In a semiclassical limit we have an equation

$$u^2 + v^2 w = 0\,. \tag{3.2}$$

Since the Coulomb branch receives quantum corrections we expect modification of the eq. (3.2).

Three-dimensional $\mathcal{N} = 4$ theory has an R-symmetry group given by $SU(2)_R \times SU(2)_N$. There are $SU(2)_R$ and $SU(2)_N$ gluino doublets, scalars A (A^+) and $\tilde{A}^+$ ($\tilde{A}$) make a doublet of $SU(2)_R$ and are singlets of $SU(2)_N$, spinors ψ ($\bar{\psi}$) and $\bar{\tilde{\psi}}$ ($\tilde{\psi}$) transform as a doublet of $SU(2)_N$ and singlets of $SU(2)_R$. Scalars χ, ϕ, and ϕ^+ form a triplet of $SU(2)_N$ and are neutral under $SU(2)_R$. In three-dimensional $\mathcal{N} = 2$ superspace formalism only the maximal torus $U(1) \times U(1)$ of the R-symmetry is manifest. Let us introduce a set of manifest R-

[3] Moduli space dimensions are assumed to be complex.

JHEP03(2004)008

symmetries denoted as $\mathrm{U}(1)_N$, $\mathrm{U}(1)_B$, and $\mathrm{U}(1)_R$ with the corresponding charges given in the table

	N	B	R
Q	0	1	1/2
$\tilde{Q}$	0	1	1/2
Φ	2	-2	1

It is easy to see that B-charge of the Grassmannian coordinates of the $\mathcal{N}=2$ superspace is zero and $R = N + \frac{1}{2}B$. The supercharge which is manifest in $\mathcal{N}=2$ superspace formalism has R-charge one, whereas a nonmanifest supercharge has vanishing R-charge.

Let us consider topology-changing operators which belong to $\mathcal{N}=4$ (Anti)BPS multiplets. In the IR limit the theory flows to the interacting superconformal field theory and (Anti)BPS representations are labelled by the (anti-)chiral primary operators. The conformal dimensions of (anti-)chiral primary operators are smaller than those of other operators in the same representation and are determined by their spin and R-symmetry representations [33, 34]. We define an (Anti)BPS monopole operator as a topology-changing operator which is an (anti-)chiral operator with a lowest conformal weight among the (anti-)chiral topology-changing operators with a given magnetic charge H. It follows that (Anti)BPS monopole operators are (anti-)chiral primaries. Using arguments similar to those presented in section 2.1 we conclude that in the large N_f limit we have matter fields in a background of (Anti)BPS monopole. Our goal will be to determine the quantum numbers of (Anti)BPS monopole operators in the limit of large N_f.

Now we will identify (Anti)BPS backgrounds corresponding to (Anti)BPS GNO monopoles in $\mathcal{N}=4$ supersymmetric gauge theory. Background values of $\vec{V}^\alpha$, ϕ^α, $\phi^{*\alpha}$, and χ^α preserve some of the manifest $\mathcal{N}=2$ supersymmetry parameterized by $\xi, \bar{\xi}$ iff they satisfy the equations

$$\delta\lambda^\alpha = -i\left(\sigma^i\left(\partial_i\chi^\alpha + f^{\alpha\beta\gamma}\chi^\beta V_i^\gamma\right) + \frac{1}{2}\epsilon^{ijk}\sigma^k V_{ij}^\alpha - D^\alpha\right)\xi = 0\,, \tag{3.3}$$

$$\delta\bar{\lambda}^\alpha = -i\bar{\xi}\left(\sigma^i\left(\partial_i\chi^\alpha + f^{\alpha\beta\gamma}\chi^\beta V_i^\gamma\right) - \frac{1}{2}\epsilon^{ijk}\sigma^k V_{ij}^\alpha + D^\alpha\right) = 0\,, \tag{3.4}$$

$$\delta\eta^\alpha = \sqrt{2}\left(f^{\alpha\beta\gamma}\chi^\beta\phi^\gamma + i\sigma^i\left(\partial_i\phi^\alpha + f^{\alpha\beta\gamma}\phi^\beta V_i^\gamma\right)\right)\bar{\xi} + \sqrt{2}\xi K^\alpha = 0\,, \tag{3.5}$$

$$\delta\bar{\eta}^\alpha = -\sqrt{2}\xi\left(f^{\alpha\beta\gamma}\chi^\beta\phi^{*\gamma} + i\sigma^i\left(\partial_i\phi^{*\alpha} + f^{\alpha\beta\gamma}\phi^{*\beta}V_i^\gamma\right)\right) + \sqrt{2}\bar{\xi}K^{*\alpha} = 0\,. \tag{3.6}$$

The other set of supersymmetry transformations is obtained from (3.3)–(3.6) by the replacements $\lambda \to \eta$, $\eta \to -\lambda$. Consider a background with $\Phi = 0$. Let us set $D^\alpha = 0$ and introduce $E_i^\alpha = -\partial_i\chi^\alpha - f^{\alpha\beta\gamma}\chi^\beta V_i^\gamma$, $B^{\alpha i} = \frac{1}{2}\epsilon^{ijk}V_{jk}^\alpha$. Equations (3.3)–(3.6) imply

$$(\vec{E}^\alpha - \vec{B}^\alpha)\vec{\sigma}\xi = 0\,, \qquad \bar{\xi}(\vec{E}^\alpha + \vec{B}^\alpha)\vec{\sigma} = 0\,.$$

For $\vec{B} = \vec{B}^\alpha T^\alpha = \frac{H}{r^3}\vec{r}$ we have the following backgrounds, each preserving half of the manifest $\mathcal{N}=2$ supersymmetry:

(i) BPS background

$$\vec{E}^\alpha = -\vec{B}^\alpha\,, \qquad \forall\bar{\xi}\,, \qquad \xi = 0\,.$$

(ii) AntiBPS background

$$\vec{E}^{\alpha} = \vec{B}^{\alpha}\,, \qquad \forall \xi\,, \qquad \bar{\xi} = 0\,.$$

We choose $\chi = \chi^{\alpha} T^{\alpha} = \mp H/r$ with $H = qT^3 = \frac{1}{2}(q, -q)$, where upper (lower) sign corresponds to the (Anti)BPS monopole backgrounds.[4] These backgrounds are invariant under $\mathrm{SU}(2)_R$ symmetry, break $\mathcal{N} = 4$ to $\mathcal{N} = 2$ supersymmetry, $\mathrm{SU}(2)_{\text{gauge}}$ group to $\mathrm{U}(1)_{\text{gauge}}$ subgroup, and $\mathrm{SU}(2)_N$ to $\mathrm{U}(1)_N$. We mention that contrary to monopoles in U(1) gauge theory, SU(2) monopoles specified by H and $-H$ are gauge equivalent.

3.2 Dual theory

The dual theory is a twisted $\mathcal{N} = 4$, $\left[\mathrm{U}(2)^{N_f - 3} \times \mathrm{U}(1)^4\right] / \mathrm{U}(1)_{\text{diag}}$ gauge theory based on the Dynkin diagram of $\mathrm{SO}(2N_f)$ group. The fields include $N_f - 3$ U(2) vector superfields which are made of $\mathcal{N} = 2$ U(1) vector superfields U_l and neutral chiral superfields T_l, SU(2) vector superfields T_l and adjoint chiral superfields S_l, $l = 1, \ldots, N_f - 3$. Also, there are four additional U(1) vector superfields which consist of $\mathcal{N} = 2$ vector superfields $U_{N_f-2}, \ldots, U_{N_f+1}$ and neutral chiral superfields $T_{N_f-2}, \ldots, T_{N_f+1}$. Factorization of the diagonal U(1) implies the constraints

$$\sum_{p=1}^{N_f+1} U_p = 0\,, \qquad \sum_{p=0}^{N_f+1} T_p = 0\,.$$

Matter fields include twisted $N_f - 4$ matter hypermultiplets made of $\mathcal{N} = 2$ chiral multiplets q_r and $\tilde{q}_r$, transforming as

$$q_r \to \mathrm{U}(2)_{r+1} q_r\, \mathrm{U}(2)^{+}_{r}\,, \qquad \tilde{q}_r \to \mathrm{U}(2)_r \tilde{q}_r\, \mathrm{U}(2)^{+}_{r+1}\,, \qquad r = 1, \ldots, N_f - 4\,.$$

We also have four additional twisted matter hypermultiplets which decompose with respect to $\mathcal{N} = 2$ as chiral superfields $(X_1, \tilde{X}_1), \ldots, (X_4, \tilde{X}_4)$. X_1 ($\tilde{X}_1$) has charge $+1$ (-1) under $\mathrm{U}(1)_{N_f-2}$ and transforms according to (anti-)fundamental representation of $\mathrm{U}(2)_1$; X_2 ($\tilde{X}_2$) has $\mathrm{U}(1)_{N_f-1}$ charge $+1$ (-1) and is belongs to (anti-)fundamental representation of $\mathrm{U}(2)_{N_f-3}$; X_3 ($\tilde{X}_3$) has a charge $+1$ (-1) under $\mathrm{U}(1)_{N_f}$ and transforms according to (anti-)fundamental representation of $\mathrm{U}(2)_1$; X_4 ($\tilde{X}_4$) has a charge $+1$ (-1) under $\mathrm{U}(1)_{N_f+1}$ and is transformed according to (anti-)fundamental representation of $\mathrm{U}(2)_{N_f-3}$. Superpotential is given by

$$\begin{aligned} W = i\sqrt{2}\Big\{ & \tilde{X}_1 \left(T_1 + S_1 - T_{N_f-2}\right) X_1 + \tilde{X}_2 \left(T_{N_f-3} + S_{N_f-3} - T_{N_f-1}\right) X_2 + \\ & + \tilde{X}_3 \left(T_1 + S_1 - T_{N_f}\right) X_3 + \tilde{X}_4 \left(T_{N_f-3} + S_{N_f-3} - T_{N_f+1}\right) X_4 + \\ & + \sum_{r=1}^{N_f-4} \tilde{q}_r \left(S_{r+1} + T_{r+1} - S_r - T_r\right) q_r \Big\}\,. \end{aligned}$$

The two dimensional Higgs branch doesn't receive quantum corrections and is given by a hyper-Kahler quotient parameterized by x, y, and z subject to a constraint

$$x^2 + y^2 z = z^{N_f - 1}\,. \tag{3.7}$$

[4] We will use this convention throughout the paper.

Explicit form of these coordinates is given in ref. [35]:

$$z = -X_1^{a_1}\tilde{X}_{3|a_1}X_3^{b_1}\tilde{X}_{1|b_1}\,, \tag{3.8}$$

and (for even N_f)

$$\begin{aligned} x &= 2X_1^{a_1}q_{1|a_1}^{a_2}\ldots q_{N_f-4|a_{N_f-4}}^{a_{N_f-3}}\tilde{X}_{2|a_{N_f-3}}X_2^{b_{N_f-3}}\tilde{q}_{N_f-4|b_{N_f-3}}^{b_{N_f-4}}\ldots\tilde{q}_{1|b_2}^{b_1}\tilde{X}_{3|b_1}X_3^{c_1}\tilde{X}_{1|c_1}\,, \\ y &= 2X_3^{a_1}q_{1|a_1}^{a_2}\ldots q_{N_f-4|a_{N_f-4}}^{a_{N_f-3}}\tilde{X}_{2|a_{N_f-3}}X_2^{b_{N_f-3}}\tilde{q}_{N_f-4|b_{N_f-3}}^{b_{N_f-4}}\ldots\tilde{q}_{1|b_2}^{b_1}\tilde{X}_{3|b_1}+(-z)^{N_f/2-1}\,. \end{aligned} \tag{3.9}$$

$4N_f-6$ dimensional Coulomb branch is parameterized by N_f+1 dual U(1) photons $V_{\pm|r}$ (for a given r, $V_{+|r}$ and $V_{-|r}$ are used as coordinates on two distinct patches) subject to the constraints

$$\prod_r V_{+|r} = \prod_r V_{-|r} = 1\,,$$

N_f independent chirals T, $2N_f-6$ independent coordinates analogous to the ones given in eq. (3.1).

3.3 Mirror symmetry

Since mirror symmetry exchanges mass and Fayet-Iliopoulos terms, we identify N_f complex mass terms $\tilde{Q}^sQ^s$ (no sum over s) with N_f independent chirals T. Therefore chirals T and S have baryon charge 2 whereas baryon charges of X, $\tilde{X}$, q, and $\tilde{q}$ are -1. Baryon charges of x, y, and z are $2-2N_f$, $4-2N_f$, and -4 respectively which can be deduced both from the defining equations (3.8)–(3.9) and from the hyper-Kahler quotient equation (3.7). Likewise, T and S have vanishing $\mathrm{U}(1)_N$ charges, whereas X, $\tilde{X}$, q, and $\tilde{q}$ have a charge $+1$. Finally, $\mathrm{U}(1)_N$ charges of x, y, and z are $2N_f-2$, $2N_f-4$, and 4 respectively. We also have $R(x)=N_f-1$, $R(y)=N_f-2$, as well as $R(z)=2$.

It follows that charges of z are independent of N_f and coincide with that of $w = 2\,\mathrm{Tr}\,\Phi^2$. Also comparing eq. (3.2) with eq. (3.7) we obtain an identification

$$u\sim x\,,\qquad v\sim y\,,\qquad w\sim z\,.$$

Thus, the mirror symmetry predicts the following charges for operators defined in eq. (3.1)

	N	B	R
u	$2N_f-2$	$2-2N_f$	N_f-1
v	$2N_f-4$	$4-2N_f$	N_f-2
w	4	-4	2

Since x, y, and z are chiral primary operators which are polynomials of the electrically charged fields, operators u, v, and w are also chiral primaries and describe the sector with nontrivial magnetic charge. As explained in ref. [20], the conformal dimension of $\mathcal{N}=4$ (anti-)chiral primary operator equals (minus) the corresponding $\mathrm{U}(1)_R$ charge.

3.4 Quantum numbers

Quantum numbers of the (Anti)BPS monopole state receive contributions from both matter hypermultiplet $\mathcal{Q}$ and vector multiplet $\mathcal{V}$. The former is proportional to N_f and is dominant in the large N_f limit, whereas the latter gives correction of the form $O(1)$. Let us determine the matter contributions first.

Energy spectra of matter fields in (Anti)BPS backgrounds are given in the appendix. Since matter fermionic particles and antiparticles have different energy spectra we adopt the "symmetric " ordering for the bilinear fermionic observables

$$\bar{\psi}O\psi \to \frac{1}{2}\bar{\psi}O\psi - \frac{1}{2}\psi O^T\bar{\psi}\,, \tag{3.10}$$

where O is some operator independent of the fields. To regulate expression on the r.h.s. of eq. (3.10) we use a substraction technique. This procedure gives

$$E^{\text{Fermions}}_{\text{Casimir}} = \frac{1}{2}\left(\sum E^- - \sum E^+\right) - \text{"the same"}|_{q=0}\,,$$

where E^+, E^- are positive and negative energies respectively. To define the formal sums appearing in this section we use

$$\sum E \to \sum E e^{-\beta|E|}$$

regularization and take $\beta \to 0$ limit at the end of calculations. Matter bosonic particles and antiparticles have identical energy spectra and standard prescription can be used. In our model the matter contribution to the Casimir energy is equal to $h_{\mathcal{Q}} = N_f|q|$ for both BPS and AntiBPS monopole backgrounds. For matter part of the R-charge operator we have

$$\begin{aligned} R_{\mathcal{Q}} = \frac{1}{4}\Big[&\sum(a^+_\psi a_\psi - a_\psi a^+_\psi) + \sum(b_\psi b^+_\psi - b^+_\psi b_\psi) + \sum(a^+_{\tilde{\psi}} a_{\tilde{\psi}} - a_{\tilde{\psi}} a^+_{\tilde{\psi}}) + \\ &+ \sum(b_{\tilde{\psi}} b^+_{\tilde{\psi}} - b^+_{\tilde{\psi}} b_{\tilde{\psi}}) - \sum(a^+_A a_A + b_A b^+_A) - \sum(a^+_{\tilde{A}} a_{\tilde{A}} + b_{\tilde{A}} b^+_{\tilde{A}})\Big] + \text{const}\,, \end{aligned}$$

where a^+ (b^+) and a (b) denote the corresponding (anti)particle creation and annihilation operators respectively. To fix a constant we define a vacuum state with zero magnetic charge $|0\rangle$ to have vanishing R-charge. It follows that

$$\langle R_{\mathcal{Q}}\rangle_q = \left(\sum_{E^-_\psi}|E^-_\psi| - \sum_{E^+_\psi}E^+_\psi + \sum_{E^+_A}E^+_A - \sum_{E^-_A}|E^-_A|\right) -'' \text{the same}''|_{q=0}\,.$$

As a result we have $\langle R_{\mathcal{Q}}\rangle = \pm h_{\mathcal{Q}}$. For N and B charges similar calculations give $\langle N_{\mathcal{Q}}\rangle_q = -\langle B_{\mathcal{Q}}\rangle_q = \pm 2N_f|q|$.

Now we will consider the vector multiplet contribution to the quantum numbers of the vacuum state. Relevant charges are summarized in the table

	N	B	R
λ	1	0	1
$\bar{\lambda}$	-1	0	-1
η	1	-2	0
$\bar{\eta}$	-1	2	0
ϕ	2	-2	1
ϕ^*	-2	2	-1
χ	0	0	0

Integration over the hypermultiplet $\mathcal{Q}$ produces an induced action for the vector multiplet $S_{\text{Ind}}[\mathcal{V}]$ proportional to N_f. Let us assume that supersymmetric monopole configuration minimizes vector multiplet effective action in the IR region. Changing $\mathcal{V} \to \mathcal{V}^{mon} + \hat{\mathcal{V}}/\sqrt{N_f}$ gives

$$S_{\text{Ind}}[\mathcal{V}] = S^{(2)}_{\text{Ind}}[\hat{\mathcal{V}}] + O\left(\frac{1}{\sqrt{N_f}}\right),$$

where $S^{(2)}_{\text{Ind}}[\hat{\mathcal{V}}]$ is quadratic in $\hat{\mathcal{V}}$ and independent from N_f. The full effective action for $\hat{\mathcal{V}}$ is

$$S_{\text{Eff}}[\mathcal{V}] = \frac{S_0[\mathcal{V}]}{e^2} + S_{\text{Ind}}[\mathcal{V}] = N_f \frac{S_0[\mathcal{V}^{\text{mon}}]}{\hat{e}^2} + \frac{S_0^{(2)}[\hat{\mathcal{V}}]}{\hat{e}^2} + S^{(2)}_{\text{Ind}}[\hat{\mathcal{V}}] + O\left(\frac{1}{\sqrt{N_f}}\right),$$

where S_0 is original action for a vector superfield and $\hat{e}^2 = e^2 N_f$. Linear term proportional to $\frac{\delta S_0}{\delta \mathcal{V}}[\mathcal{V}^{\text{mon}}]\,\hat{\mathcal{V}}$ vanishes because supersymmetric field configuration $\mathcal{V}^{\text{mon}}$ automatically minimizes the action S_0.

The superconformal algebra arising in the IR limit has generators $\mathbb{S}$ and $\bar{\mathbb{S}}$ which are superpartners of the special conformal transformations $\mathbb{K}$:

$$[\mathbb{K}, \mathbb{Q}] \sim \bar{\mathbb{S}}, \qquad [\mathbb{K}, \bar{\mathbb{Q}}] \sim \mathbb{S}.$$

In the euclidean space we have $\mathbb{Q}^+ = \mathbb{S}$ and $\bar{\mathbb{Q}}^+ = \bar{\mathbb{S}}$, hence special conformal transformations generated by $\bar{\mathbb{S}}$ ($\mathbb{S}$) leave the (Anti)BPS background invariant. The (Anti)BPS background breaks some of the global symmetries, and the full Hilbert space of states is given by a tensor product of the physical Fock space constructed from a vacuum state and a space of superfunctions on the appropriate supercoset [20]. In ref. [34] it was shown that unitarity condition applied to the anticommutator $\{\bar{\mathbb{Q}}, \bar{\mathbb{S}}\}$ in the $\mathcal{N} = 2$ supersymmetric theory implies that the conformal weight h and infrared R-charge R_{IR} of any state satisfy $h \geq R_{IR}$. Also, it follows from the anticommutator $\{\mathbb{Q}, \mathbb{S}\}$ that $h \geq -R_{IR}$ in a unitary theory. As explained in ref. [20], the infrared R-charge R_{IR} coincides with $\mathrm{U}(1)_R$ charge R. Thus for the physical Fock space constructed from the (Anti)BPS vacuum, the unbroken subalgebra of the three-dimensional superconformal algebra implies that (minus) R-charge of a state can not exceed its conformal dimension:

$$h \geq \pm R, \tag{3.11}$$

with the lower bound saturated by the (anti-)chiral primary operators with vanishing spin.

Let us focus on the gluino contribution to the R-charge. We have two sets of gluinos $\hat\lambda$ and $\hat\eta$. Since $\mathrm{U}(1)_R$ symmetry acts trivially on $\hat\eta$, the only contribution comes from $\hat\lambda$. Relevant quadratic terms in the effective action have the form (in R^3):

$$S_0^{(2)}[\hat\lambda,\hat{\bar\lambda}] = \int dx \Bigg(i\hat{\bar\lambda}_+ \left[\vec\sigma\left(\vec\nabla - i\vec V^{\mathrm{mon}}\right) \pm \frac{q}{r}\right]\hat\lambda_+ + i\hat{\bar\lambda}_-\left[\vec\sigma\left(\vec\nabla + i\vec V\right) \mp \frac{q}{r}\right]\hat\lambda_- + \\ + i\hat{\bar\lambda}^3\vec\sigma\vec\nabla\hat\lambda^3\Bigg)\,,$$

$$S_{\mathrm{Ind}}^{(2)}[\hat\lambda,\hat{\bar\lambda}] = \int dxdy \Bigg(\hat{\bar\lambda}_+(x)O^{(+)}(x,y)\hat\lambda_+(y) + \hat{\bar\lambda}_-(x)O^{(-)}(x,y)\hat\lambda_-(y) \\ + \hat{\bar\lambda}^3(x)O^{(3)}(x,y)\hat\lambda^3(y)\Bigg)\,,$$

with $\hat\lambda_+ = (\hat\lambda^1 + i\hat\lambda^2)/\sqrt2$, $\hat\lambda_- = (\hat\lambda^1 - i\hat\lambda^2)/\sqrt2$, and

$$O^{(+)}(x,y) \sim \left\langle\bar\psi_1(x)\psi_1(y)\right\rangle\left\langle A_2(x)A_2^+(y)\right\rangle, \qquad O^{(-)}(x,y) \sim \left\langle\bar\psi_2(x)\psi_2(y)\right\rangle\left\langle A_1(x)A_1^+(y)\right\rangle,$$
$$O^{(3)}(x,y) \sim \left\langle\bar\psi_1(x)\psi_1(y)\right\rangle\left\langle A_1(x)A_1^+(y)\right\rangle + \left\langle\bar\psi_2(x)\psi_2(y)\right\rangle\left\langle A_2(x)A_2^+(y)\right\rangle,$$

where we used the identities

$$\left\langle\bar{\tilde\psi}_1(x)\tilde\psi_1(y)\right\rangle = \left\langle\bar\psi_2(x)\psi_2(y)\right\rangle, \qquad \left\langle\bar{\tilde\psi}_2(x)\tilde\psi_2(y)\right\rangle = \left\langle\bar\psi_1(x)\psi_1(y)\right\rangle,$$
$$\left\langle\tilde A_1(x)\tilde A_1^+(y)\right\rangle = \left\langle A_2(x)A_2^+(y)\right\rangle, \qquad \left\langle\tilde A_2(x)\tilde A_2^+(y)\right\rangle = \left\langle A_1(x)A_1^+(y)\right\rangle.$$

R-charge contribution of $\hat\lambda_+$ and $\hat{\bar\lambda}_+$ can be expressed in terms of η-invariant of the hamiltonian associated with $O^{(+)}$. If $\hat\lambda_+$ has zero-energy modes in the Fock space, it may lead to ambiguities in the R-charge computation. Let us show that such modes are not present. Induced action equation of motion $\delta S_{\mathrm{Ind}}^{(2)}/\delta\hat{\bar\lambda}_+ = 0$ has the form

$$\int dy O^{(+)}(x,y)\hat\lambda_+(y) = 0\,. \tag{3.12}$$

Transforming to $S^2\times R$ and assuming $\hat\lambda_+$ independent of τ, we obtain

$$\int d\tau_y d\varphi_y d\theta_y O^{(+)}(\varphi_x,\theta_x,\tau_x;\varphi_y,\theta_y,\tau_y)\hat\lambda_+(\varphi_y,\theta_y) = 0\,. \tag{3.13}$$

If eq. (3.13) has a non-trivial solution corresponding to an operator acting in the Fock space, $\mathrm{SU}(2)_R$ symmetry implies that $\hat\eta_+ = (\hat\eta_1 + i\hat\eta_2)/\sqrt2$ also has a zero-energy mode. Then it follows from the supersymmetry transformation

$$\delta\hat\phi_+^* = \sqrt2\bar\xi\hat{\bar\eta}_+e^{-\tau/2}\,,$$

that $\hat\phi_+$ has a mode with energy $-1/2$ in the Fock space associated with BPS monopole background. Let us denote the corresponding creation operator as $b_{\hat\phi_+}^{+\{|E^-|=1/2\}}$. Using the explicit form of the matter field energy modes it is straightforward to check that

$$O^{(-)} = O^{(+)}|_{\varphi_x\to-\varphi_x,\varphi_y\to-\varphi_y}\,,$$

which implies that there is a zero-energy solution for $\hat\lambda_-$ as well. Hence, $\hat\eta_-$ has zero-energy mode and $\hat\phi_-$ has a mode with energy $-1/2$ which we denote as $b_{\hat\phi_-}^{+\{|E^-|=1/2\}}$. The product $b^{+\{|E^-|=1/2\}}_{\hat{\phi}_+} b^{+\{|E^-|=1/2\}}_{\hat{\phi}_-}$ is $\mathrm{U}(1)_{\mathrm{gauge}}$ invariant operator which has R-charge 2 and energy (conformal dimension) 1. Repeated action of this operator on any physical state with definite R-charge and conformal dimension will finally give a state with R-charge greater than the conformal dimension which violates the unitarity bound (3.11). Thus we conclude that $\hat{\lambda}_+$ does not have zero-energy modes in the Fock space constructed from the BPS vacuum. For AntiBPS monopole background similar analysis gives analogous conclusion.

Action $S^{(2)}_{\mathrm{Eff}}$ acquires explicit τ-dependence on $S^2 \times R$ as a reminiscence of the fact that theory is not conformal invariant for $0 < \hat{e}^2 < \infty$. Let us define $g^2 = e^{\tau}\hat{e}^2$ and consider the resulting theory $S^{(2)}_g$, which can be viewed as conformal invariant deformation of $S^{(2)}_{\mathrm{Ind}}$ with constant g being a deformation parameter. Let $\{E_k(g)\}$ be the energy spectrum of $\hat{\lambda}_+$, then the R-charge contribution is

$$\left\langle R_{\hat{\lambda}_+,\hat{\bar{\lambda}}_+} \right\rangle_q = \lim_{\beta\to 0} \left(Z(q,\beta) - Z(0,\beta)\right), \qquad Z(q,\beta) = \frac{1}{2}\sum_k \mathrm{sign}\left[E_k\left(g\right)\right] e^{-\beta|E_k(g)|}\,. \quad (3.14)$$

Since $Z(q,\beta)$ is proportional to η-invariant, $\langle R_{\hat{\lambda}_+,\hat{\bar{\lambda}}_+}\rangle_q$ is expected to be independent from g and, hence, can be computed in the region of small g. To make this argument rigorous it is necessary to show that $\hat{\lambda}_+$ does not have zero-energy modes for all values of the constant g. We hope to return to this problem in the future. If g is small the induced action terms can be ignored and we have gluinos moving in a monopole background $\mathcal{V}^{\mathrm{mon}}$. The hamiltonian eigen-value equation has a form

$$\left(\mathcal{H}_{\psi_2}|_{q\to 2q} - \frac{1}{2}\right)\hat{\lambda}_+ = E\hat{\lambda}_+\,,$$

where $\mathcal{H}_{\psi_2}$ is hamiltonian for the matter field ψ_2. Using energy spectrum of ψ_2 given in the appendix, we find that the spectrum of $\hat{\lambda}_+$ in the limit of small g is given by ($n = 1, 2, \ldots$)

$$E = -|q| - n - \frac{1}{2}\,, \qquad \mp|q| - \frac{1}{2}\,, \qquad |q| + n - \frac{1}{2}\,,$$

where each energy level has degeneracy $|2E+1|$. We mention that energy level $E = \mp|q| - \frac{1}{2}$ has degeneracy $2|q|$ and is not present if $q = 0$. Using eq. (3.14) in the small g region we obtain $< R_{\hat{\lambda}_+,\hat{\bar{\lambda}}_+} >_q = \mp|q|$.

Similar analysis can be implemented for $\hat{\lambda}_-$ and $\hat{\lambda}^3$. R-charge contribution of $\hat{\lambda}_-$ is identical to that of $\hat{\lambda}_+$, whereas in the small g limit $\hat{\lambda}^3$ is moving in the trivial ($\mathcal{V} = 0$) background and does not contribute to the R-charge. Besides gluinos $\hat{\lambda}$, the only vector multiplet fields charged under the $\mathrm{U}(1)_R$ symmetry are scalars ϕ and ϕ^*. Analogous calculations show that they do not contribute to the $O(1)$ terms of the R-charge. Therefore, in the large N_f limit, we have

$$\langle R\rangle_q = \pm\left(N_f - 2\right)|q|\,.$$

For the N-charge we have $\langle N_{\mathcal{V}}\rangle_q =< N_{\hat{\lambda},\hat{\bar{\lambda}}} >_q + < N_{\hat{\eta},\hat{\bar{\eta}}} >_q = 2 < N_{\hat{\lambda},\hat{\bar{\lambda}}} >_q$, where we used invariance of the (Anti)BPS background under $\mathrm{SU}(2)_R$:

$$S^{(2)}_{\mathrm{Ind}}[\hat{\eta},\hat{\bar{\eta}}] = S^{(2)}_{\mathrm{Ind}}[\hat{\lambda},\hat{\bar{\lambda}}]\Big|_{\hat{\lambda}\to\hat{\eta},\hat{\bar{\lambda}}\to\hat{\bar{\eta}}}\,.$$

JHEP03(2004)008

Calculations similar to those for the R-charge give $\langle N_{\mathcal{V}}\rangle_q = \mp 4|q|$, which implies

$$\langle N\rangle_q = \pm\left(2N_f - 4\right)|q|, \quad \langle B\rangle_q = \pm(4 - 2N_f)|q|\,.$$

3.5 Comparison with the mirror symmetry predictions

Mirror symmetry implications for the quantum numbers of w (see eq. (3.1)) are trivially satisfied. Thus we conclude that $w \sim z$. Let us consider a physical state $|vac\rangle_q$. It is the lowest energy state and, therefore, a superconformal primary. Since (Anti)BPS background is annihilated by a supercharge $\bar{\mathbb{Q}}$ $(\mathbb{Q})$, a state $\bar{\mathbb{Q}}\,|vac\rangle_q$ $\left(\mathbb{Q}|\,vac\rangle_q\right)$ belongs to the Fock space associated with $|vac\rangle_q$. Since supercharge $\bar{\mathbb{Q}}$ $(\mathbb{Q})$ raises energy by $1/2$ and has $\mathrm{U}(1)_R$ charge (minus) one, we find that there is no such a physical state in the Fock space. Therefore, a state $|vac\rangle_q$ is annihilated by $\bar{\mathbb{Q}}$ $(\mathbb{Q})$ and corresponds to the insertion of the (anti-)chiral primary operator at the origin of R^3. Thus, conformal weight of $|vac\rangle_q$ equals its R-charge, i.e., $\pm\left(N_f - 2\right)|q|$. Background $\Phi = 0$ also corresponds to the (Anti)BPS monopoles in $\mathcal{N} = 2$ SU(2) gauge theory which can be obtained by giving mass to the adjoint chiral field Φ. Therefore, matter contribution to the quantum numbers of the $\mathcal{N} = 2$ (Anti)BPS monopoles is the same as in the $\mathcal{N} = 4$ theory. We also note that chiral primaries u and w are present for $\mathcal{N} = 4$ only and absent in $\mathcal{N} = 2$ theory. This observation implies

$$|vac\rangle^{\mathrm{BPS}}_{|q|=1} \propto v(0)\,|0\rangle\,, \qquad |vac\rangle^{\mathrm{AntiBPS}}_{|q|=1} \propto v^{+}(0)\,|0\rangle\,.$$

Identity of $|vac\rangle^{\mathrm{BPS}}_{|q|=1}$ and y quantum numbers gives $v \sim y$.

To obtain another (anti-)chiral primary state in the physical Fock space, we must act on a state $|vac\rangle^{\mathrm{(Anti)BPS}}_{|q|=1}$ with an $\mathrm{U}(1)_{\mathrm{gauge}}$ invariant operator f such that it raises energy by $R(f)$ $(-R(f))$. It is easy to see that f can not be made of matter fields only. Indeed, the most general expression for (anti-)chiral primary $f\left(\mathcal{Q}\right)|vac\rangle_q$ would be a superposition of gauge invariant states of the form $\left(a^{+}_{\mathcal{Q}}\right)^m \left(b^{+}_{\mathcal{Q}}\right)^p |vac\rangle_q$ with some non-negative integers m and p. However,

$$E\left(a^{+}_{\mathcal{Q}}\right) > \pm R\left(a^{+}_{\mathcal{Q}}\right), \qquad E\left(b^{+}_{\mathcal{Q}}\right) > \pm R\left(b^{+}_{\mathcal{Q}}\right),$$

and the state $\left(a^{+}_{\mathcal{Q}}\right)^m \left(b^{+}_{\mathcal{Q}}\right)^p |vac\rangle_q$ is not an (anti-)chiral primary, unless $m = p = 0$.

Now we consider energy spectra of fields which belong to the vector multiplet. In the IR limit the only terms in the vector multiplet effective action are those induced by integration over the matter hypermultiplets. Let us show that gluinos $\hat\eta$, $\hat{\bar\eta}$, $\hat\lambda$, $\hat{\bar\lambda}$ do not have (anti-)chiral primary creation operators in the Fock space associated with the (Anti)BPS background. It follows from eq. (3.11) that such modes can not be present in $\hat{\bar\eta}$ and $\hat{\bar\lambda}$, ($\hat\eta$ and $\hat\lambda$). Since R-charge of $\hat\eta$ vanishes, the (anti-)chiral primary creation operator corresponds to a mode with zero energy. It was shown in section 3.4 that $\hat\eta_+$ and $\hat\eta_-$, ($\hat{\bar\eta}_+$ and $\hat{\bar\eta}_-$), do not have zero-energy modes. If a gauge invariant field $\hat\eta^{(3)}$ $(\hat{\bar\eta}^{(3)})$ has a creation operator with zero energy, then $\mathrm{SU}(2)_R$ symmetry implies existence of $b^{+\{E=0\}}_{\hat\lambda^{(3)}}$, $(a^{+\{E=0\}}_{\hat\lambda^{(3)}})$, which is incompatable with eq. (3.11). If present, an (anti-)chiral primary creation operator of $\hat\lambda^\alpha$ $(\hat{\bar\lambda}^\alpha)$ has the form $b^{+\{|E^-|=1\}}_{\hat\lambda^\alpha}$, $(a^{+\{|E^-|=1\}}_{\hat{\bar\lambda}^\alpha})$. Then $\mathrm{SU}(2)_R$ symmetry ensures existence of $b^{+\{|E^-|=1\}}_{\hat\eta^\alpha}$, $(a^{+\{|E^-|=1\}}_{\hat\eta^\alpha})$. The supersymmetry transformation $\delta\hat\phi^{*\alpha} =$

$\sqrt{2}\bar{\xi}\hat{\bar{\eta}}^{\alpha}e^{-\tau/2}$, ($\delta\hat{\phi}^{\alpha} = \sqrt{2}\xi\hat{\eta}^{\alpha}e^{\tau/2}$), implies a presence of $\hat{\phi}^{\alpha}$ mode with energy $|E^-| = 3/2$: $\hat{\phi}^{\alpha} \sim e^{3\tau/2}$, $\hat{\phi}^{*\alpha} \sim e^{-3\tau/2}$. Such modes should annihilate the right-hand-side of $S^2 \times R$ counterpart of eq. (3.5), (eq. (3.6)), for all $\bar{\xi}$ (ξ) in the (Anti)BPS monopole background to ensure that an operator $b^{+\{|E^-|=1\}}_{\hat{\eta}^{\alpha}}$, ($a^{+\{|E^-|=1\}}_{\hat{\eta}^{\alpha}}$) is annihilated by $\bar{\mathbb{Q}}$ ($\mathbb{Q}$). It is easy to see that it can not be the case. Thus we conclude that gluinos do not have (anti-)chiral primary creation operators in the Fock space. Similar arguments reveal that it is true for $\hat{\chi}^{\alpha}$ and $\hat{V}^{\alpha}_i$ as well. Thus $\hat{\phi}$ and $\hat{\phi}^*$ are the only fields which could have such modes.

It follows from eq. (3.11) that energy spectrum of $\hat{\phi}^{\alpha}$ satisfies $|E^-| \geq R(\hat{\phi}^{\alpha}) = 1$, ($E^+ \geq -R(\hat{\phi}^{*\alpha}) = 1$). The (Anti)BPS background under consideration has vanishing expectation values of $\mathrm{U}(1)_{\mathrm{gauge}}$ invariant fields $\phi^{(3)}$ and $\phi^{*(3)}$. However, as it follows from eqs. (3.3)–(3.6), setting $\phi^{(3)} = c$, ($\phi^{*(3)} = c$),[5] with constant c in R^3 leaves the (Anti)BPS background invariant under $\bar{\mathbb{Q}}$ ($\mathbb{Q}$). Therefore, the action $S_{\mathrm{Eff}}[\mathcal{V}]$ is stationary on these field configurations. Since the constant c is arbitrary, quadratic part of $S_{\mathrm{Eff}}[\hat{\mathcal{V}}]$ is stationary as well. In the IR limit it implies existence of the creation operator $b^{+\{|E^-|=1\}}_{\hat{\phi}^{(3)}}$ ($a^{+\{E^+=1\}}_{\hat{\phi}^{(3)}}$), corresponding to the spinless mode of $\hat{\phi}^{(3)}$ ($\hat{\phi}^{*(3)}$) on $S^2 \times R$. In the (Anti)BPS background any creation operator of $\hat{\phi}^{(3)}$ ($\hat{\phi}^{*(3)}$) corresponding to a mode with energy $|E^-| = 1$ ($E^+ = 1$) saturates the unitarity bound given by eq. (3.11). Hence, this mode has vanishing spin and is given by $const \times e^{\tau}$ on $S^2 \times R$. Thus the (anti-)chiral primary mode of $\hat{\phi}^{(3)}$ ($\hat{\phi}^{*(3)}$) in the (Anti)BPS background is unique. Acting with the corresponding creation operators on the state $|vac\rangle^{\mathrm{(Anti)BPS}}_{|q|=1}$ we obtain chiral primaries with the quantum numbers identical to those predicted for u (u^+). We have

$$b^{+\{|E^-|=1\}}_{\hat{\phi}^{(3)}}\,|vac\rangle^{\mathrm{BPS}}_{|q|=1} \propto u(0)\,|0\rangle\,, \qquad a^{+\{E^+=1\}}_{\hat{\phi}^{(3)}}\,|vac\rangle^{\mathrm{AntiBPS}}_{|q|=1} \propto u^+(0)\,|0\rangle\,.$$

The BPS background breaks the Weyl subgroup Z_2 spontaneously and Z_2 invariance of the physical states is not required. However, it might be instructive to construct Z_2 invariant (anti-)chiral primary states by "integrating" the physical states over Z_2. Let us introduce a pair of gauge equivalent states

$$|vac\rangle^{\mathrm{BPS}}_{q=1} \propto Y_+\,|0\rangle\,, \qquad |vac\rangle^{\mathrm{BPS}}_{q=-1} \propto Y_-\,|0\rangle\,.$$

Then,

$$v(0)\,|0\rangle \propto |vac\rangle^{\mathrm{BPS}}_{q=1} + |vac\rangle^{\mathrm{BPS}}_{q=-1}\,, \qquad u(0)\,|0\rangle \propto \phi^{(3)}\left(|vac\rangle^{\mathrm{BPS}}_{q=1} - |vac\rangle^{\mathrm{BPS}}_{q=-1}\right).$$

Similar construction can be made for the AntiBPS monopole operators as well.

4. Discussion

We have studied monopole operators in non-supersymmetric $\mathrm{SU}(N_c)$ gauge theories as well as (Anti)BPS monopole operators in $\mathcal{N} = 4$ SU(2) gauge theories in the limit of large N_f. In the case of $\mathrm{SU}(N_c)$ non-supersymmetric gauge theories we found that monopole operators with minimum magnetic charge have zero spin and transform nontrivially under the flavor symmetry group. Conformal dimensions of these operators have leading terms of the order N_f and further sub-leading corrections are expected.

[5]It also implies setting $\phi^{*(3)}$ ($\phi^{(3)}$) to c^*/r^2 in R^3.

JHEP03(2004)008

In the case of $\mathcal{N}=4$ SU(2) gauge theory, the mirror symmetry predicts existence of two (anti-)chiral primary monopole operators corresponding to the (anti-)chiral primary operators x (x^+) and y (y^+) in the dual theory. The (anti-)chiral primary operator dual to y (y^+) exists in $\mathcal{N}=2$ theory as well, whereas existence of the (anti-)chiral primary dual to x (x^+) is a special feature of $\mathcal{N}=4$ theory. Using the radial quantization we have shown that a state $|vac\rangle^{\text{(Anti)BPS}}_{|q|=1}$ corresponds to the insertion of the (anti-)chiral primary monopole operator which is dual to the operator y (y^+) in the large N_f limit. We demonstrated that there is unique (anti-)chiral primary monopole operator with quantum numbers matching those of x (x^+). However we note that the relation in the chiral ring implied by eq. (3.7) remains obscure.

We have shown that (Anti)BPS monopole operators of $\mathcal{N}=4$ supersymmetric theory are scalars under the $\mathrm{SU}(2)_{\mathrm{rot}}$ and transform trivially under the flavor symmetry group. Transformation properties under the global symmetries have been computed in the large N_f limit providing a new nontrivial verification of three-dimensional mirror symmetry. Although we perform calculations using $1/N_f$ expansion, our result for quantum numbers of the (Anti)BPS monopole operators are exact and do not receive $O(1/N_f)$ corrections. The reason is that the charges which correspond to U(1) subgroups of the compact R-symmetry group must be integral.

It might be interesting to generalize the analysis of the present paper to the monopole operators of $\mathcal{N}=4$ SU(N_c) gauge theories with $N_c>2$.

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A. Radial quantization of three-dimensional $\mathcal{N}=4$ SU(2) gauge theory

We start with $\mathcal{N}=2$ lagrangian density in four dimensional Minkowski space[6] for the vector multiplet $\mathcal{V}$ in the adjoint representation of SU(2) and hypermultiplets $\mathcal{Q}^s$ in the fundamental representation of the gauge group

$$\begin{aligned}\mathcal{L}^{R^{3,1}}_{\mathcal{V}} &= \frac{1}{8e^2}\left(\int d^2\theta\ \mathrm{Tr}\,(W^\alpha W_\alpha)|_{\bar\theta=0}+\text{h.c.}\right)+\frac{1}{e^2}\int d^2\theta d^2\bar\theta\,\mathrm{Tr}(\Phi^+e^{2V}\Phi)\,,\\ \mathcal{L}^{R^{3,1}}_{\mathcal{Q}} &= \int d^2\theta d^2\bar\theta\sum_{s=1}^{N_f}\left(Q^{s+}e^{2V}Q^s+\tilde Q^s e^{-2V}\tilde Q^{s+}\right)+\left(\int d^2\theta W+\text{h.c.}\right),\end{aligned}$$

where a superpotential $W=i\sqrt{2}\sum_{s=1}^{N_f}\tilde Q^s\Phi Q^s$. Let us perform the Wick rotation to R^4

$$\mathcal{L}_{R^4}=-\mathcal{L}_{R^{3,1}}|_{x^0=-it}\,,\qquad V_0^\alpha|_{R^{3,1}}=i\chi^\alpha|_{R^4}\,,$$

[6] We adopt the notations of Wess and Bagger, ref. [36].

and assume that all fields are independent of the euclidean time t. This procedure gives $\mathcal{N}=4$ supersymmetric lagrangian density in three-dimensional euclidean space:

$$
\begin{aligned}
\mathcal{L}_Q^{R^3} &= i\bar{\psi}\vec{\sigma}\left(\vec{\nabla}+i\vec{V}\right)\psi + i\bar{\psi}\chi\psi + \left(\left[\vec{\nabla}+i\vec{V}\right]A\right)^+\left(\left[\vec{\nabla}+i\vec{V}\right]A\right) + A^+\chi^2 A + \\
&\quad + i\sqrt{2}\left(\bar{\psi}\bar{\lambda}A - A^+\lambda\psi\right) - F^+F - A^+DA + i\bar{\tilde{\psi}}\vec{\sigma}(\vec{\nabla}-i\vec{V}^T)\tilde{\psi} - i\bar{\tilde{\psi}}\chi^T\tilde{\psi} + \\
&\quad + \left(\left[\vec{\nabla}+i\vec{V}\right]\tilde{A}^+\right)^+\left(\left[\vec{\nabla}+i\vec{V}\right]\tilde{A}^+\right) + \tilde{A}\chi^2\tilde{A}^+ - \\
&\quad - \tilde{F}\tilde{F}^+ + \tilde{A}D\tilde{A}^+ - i\sqrt{2}\left(\tilde{A}\bar{\lambda}\bar{\tilde{\psi}} - \tilde{\psi}\lambda\tilde{A}^+\right) + \cdots ,
\end{aligned}
$$

where the dots denote terms originated from the superpotential and summations over color and flavor indices are implied. To obtain a theory on $S^2\times R$ we perform the Weyl rescaling $g_{ij}\to r^2 g_{ij}$ and introduce $\tau=\ln r$. The matter fields transform as

$$
\left(\psi,\bar{\psi},\tilde{\psi},\bar{\tilde{\psi}}\right)\to e^{-\tau}\left(\psi,\bar{\psi},\tilde{\psi},\bar{\tilde{\psi}}\right), \qquad \left(A,A^+,\tilde{A},\tilde{A}^+\right)\to e^{-\frac{\tau}{2}}\left(A,A^+,\tilde{A},\tilde{A}^+\right).
$$

For fields in the vector multiplet we have

$$
\left(\chi,\phi,\phi^+\right)\to e^{-\tau}\left(\chi,\phi,\phi^+\right), \qquad \vec{V}\to\vec{V}, \qquad \left(\lambda,\bar{\lambda},\eta,\bar{\eta}\right)\to e^{-\frac{3}{2}\tau}\left(\lambda,\bar{\lambda},\eta,\bar{\eta}\right).
$$

The (Anti)BPS background is diagonal in color indices and, therefore, we may use results of ref. [20] for matter energy spectra in a background of U(1) monopole with a substitution $q\to q/2$. Solutions with energy E have the form $Q,\tilde{Q}\sim e^{-E\tau}$, whereas $Q^+,\tilde{Q}^+\sim e^{E\tau}$. To summarize we have, ($n=1,2,\dots$):

$$
E=-\frac{|q|}{2}-n\,, \qquad \mp\frac{|q|}{2}, \qquad \frac{|q|}{2}+n\,, \tag{A.1}
$$

for ψ_a^s, $\tilde{\psi}_a^s$ and

$$
E=-\frac{|q|}{2}-n\,, \qquad \pm\frac{|q|}{2}, \qquad \frac{|q|}{2}+n\,,
$$

for $\bar{\psi}_a^s$ and $\bar{\tilde{\psi}}_a^s$. Scalar fields A_a^s, $\tilde{A}_a^s$, A_a^{s+}, and $\tilde{A}_a^{s+}$ have

$$
E=-\frac{|q|-1}{2}-n\,, \qquad \frac{|q|-1}{2}+n\,.
$$

Each energy level with energy E has a spin $j=|E|-1/2$ and a degeneracy $2|E|$. We notice that fermionic spectrum is not invariant under $E\to -E$. The fact that A and $\tilde{A}^+$ have identical energy spectra is consistent with the action of $\mathrm{SU}(2)_R$ symmetry. On the other hand fields ψ and $\bar{\tilde{\psi}}$ have different energy spectra which conforms with the breaking of $\mathrm{SU}(2)_N$ symmetry to a $\mathrm{U}(1)_N$ subgroup which doesn't mix these fermionic fields.

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